ELECTROSTATIC POTENTIAL

When an external force does work in taking a body from a point to another against a force like spring force or gravitational force,
that work gets stored as potential energy of the body. The sum of kinetic and potential energies is thus conserved

The work done per unit test charge is characteristic of the electric field associated with the charge configuration.

Work done by external force in bringing a unit positive charge from point R to P $= V_P - V_R$

V_P is the Electrostatic potential at P and

V_R is the Electrostatic potential at P and

If U_p is the work done in moving a charge q from infinity to the point, then

$$V_p = U_p/q$$

Similarly $V_R = U_R/q$

Work done by external force in bringing a unit positive charge from point R to P

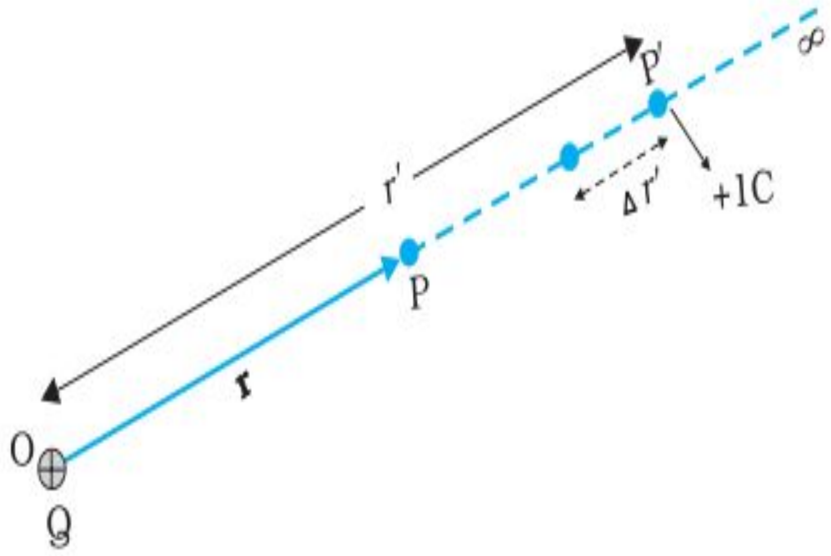
$$= V_P - V_R \left(= \frac{U_P - U_R}{q} \right)$$

Work done by an external force in bringing a unit positive charge from infinity to a point = electrostatic potential (V) at that point.

In other words, the **electrostatic potential** (V) at any point in a region with electrostatic field is

the **work done** in bringing a **unit positive** charge (without acceleration) from **infinity** to **that** point.

POTENTIAL DUE TO A POINT CHARGE



To find the **potential**, calculate the work done in bringing a unit positive test charge from **infinity** to the **point P**.

At some intermediate point P′ on the path, the
electrostatic force on a unit posit

$$\frac{Q \times 1}{4\pi\varepsilon_0 r'^2}\hat{\mathbf{r}}'$$

where r ′ is the unit vector
along OP′.

Work done against this
force from r′ to r′ + Δr′ is

$$\Delta W = -\frac{Q}{4\pi\varepsilon_0 r'^2}\Delta r'$$

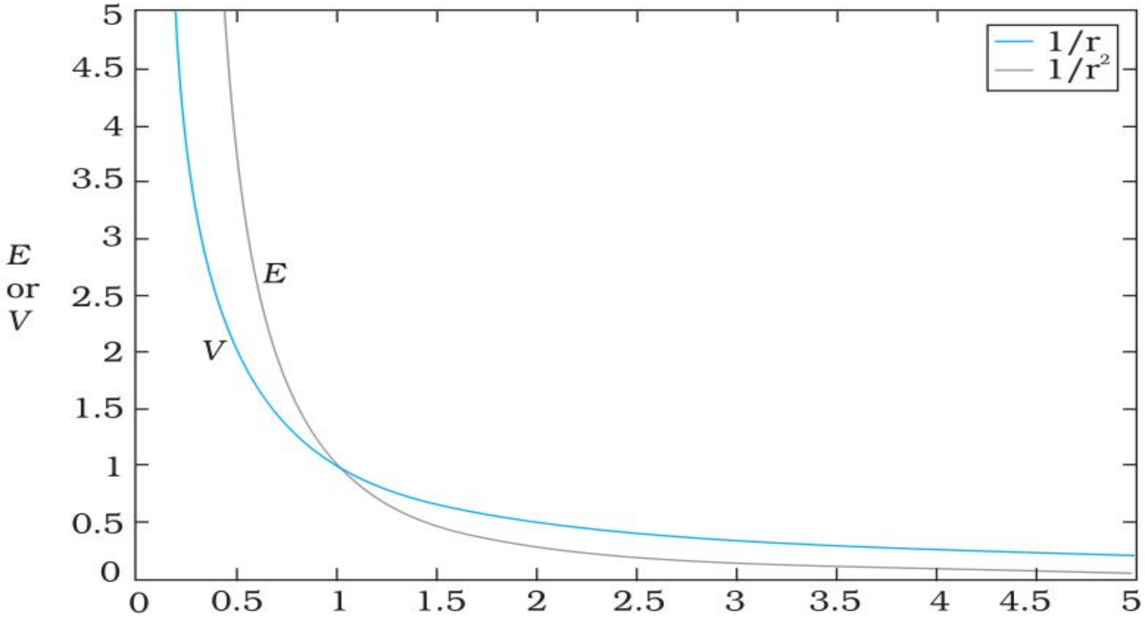
The negative sign appears because for $\Delta r' < 0$, ΔW is positive . Total work done (W) by the external force is obtained by

$$W = -\int_{\infty}^{r} \frac{Q}{4\pi\varepsilon_0 r'^2} dr' = \frac{Q}{4\pi\varepsilon_0 r'}\Bigg|_{\infty}^{r} = \frac{Q}{4\pi\varepsilon_0 r}$$

This, by definition is the potential at P due to the charge Q

$$V(r) = \frac{Q}{4\pi\varepsilon_0 r}$$

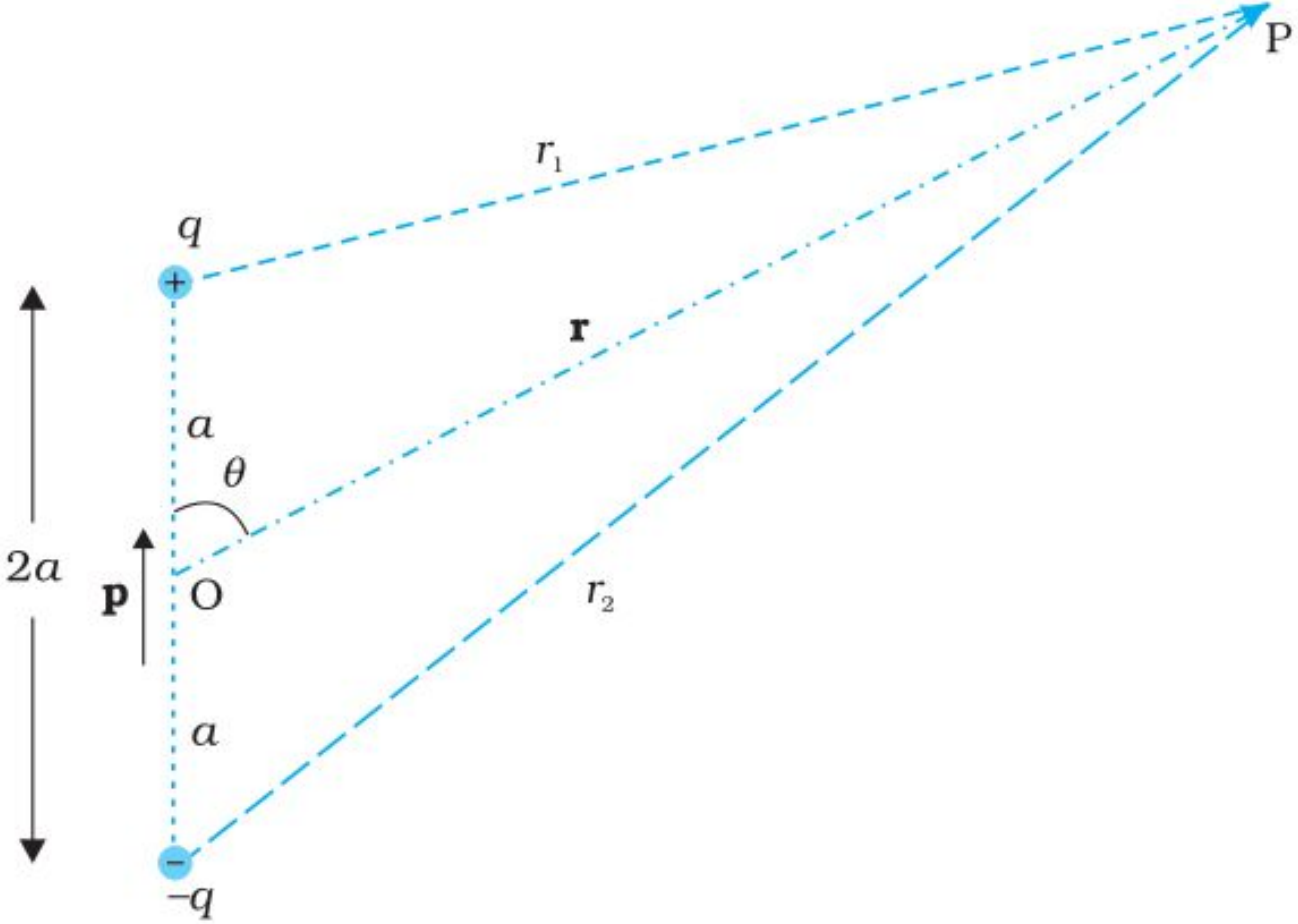
The electrostatic potential is ∝ 1/r and the electrostatic field is ∝ $1/r^2$



POTENTIAL DUE TO AN ELECTRIC DIPOLE

The electric field of a dipole at a point with position vector r depends not just on the **magnitude r**, but also on the **angle** between **r** and **p**.

Since potential is related to the work done by the field, electrostatic potential also follows the **superposition** principle.


P
r_1
q
$\mathbf{r}$
a
θ
$2a$
$\mathbf{p}$
O
r_2
a
$-q$

Thus, the potential due to the dipole is the sum of potentials due to the charges q and –q

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_1} - \frac{q}{r_2}\right)$$

where r_1 and r_2 are the distances of the point P from q and –q, respectively.

Now, by geometry,

$r_1^2 = r^2 + a^2 - 2ar\cos\theta$

$r_2^2 = r^2 + a^2 + 2ar\cos\theta$

We take r much greater than a (r >> a) and retain terms only upto the first order in a/r

$$r_1^2 = r^2\left(1 - \frac{2a\cos\theta}{r} + \frac{a^2}{r^2}\right) \quad \cong r^2\left(1 - \frac{2a\cos\theta}{r}\right)$$

Similarly,

$$r_2^2 \cong r^2 \left(1 + \frac{2a\cos\theta}{r}\right)$$

Using the Binomial theorem and retaining terms upto the first order in a/r ; we obtain,

$$\frac{1}{r_1} \cong \frac{1}{r}\left(1-\frac{2a\cos\theta}{r}\right)^{-1/2} \cong \frac{1}{r}\left(1+\frac{a}{r}\cos\theta\right)$$

$$\frac{1}{r_2} \cong \frac{1}{r}\left(1+\frac{2a\cos\theta}{r}\right)^{-1/2} \cong \frac{1}{r}\left(1-\frac{a}{r}\cos\theta\right)$$

$$V = \frac{q}{4\pi\varepsilon_0}\frac{2a\cos\theta}{r^2} = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$$

Now, $p\cos\theta = \mathbf{p}\cdot\hat{\mathbf{r}}$

where r is the unit vector along the position vector OP.

The electric potential of a dipole is then given by

$$V = \frac{1}{4\pi\varepsilon_0}\frac{\mathbf{p}\cdot\hat{\mathbf{r}}}{r^2}; \qquad (r >> a)$$

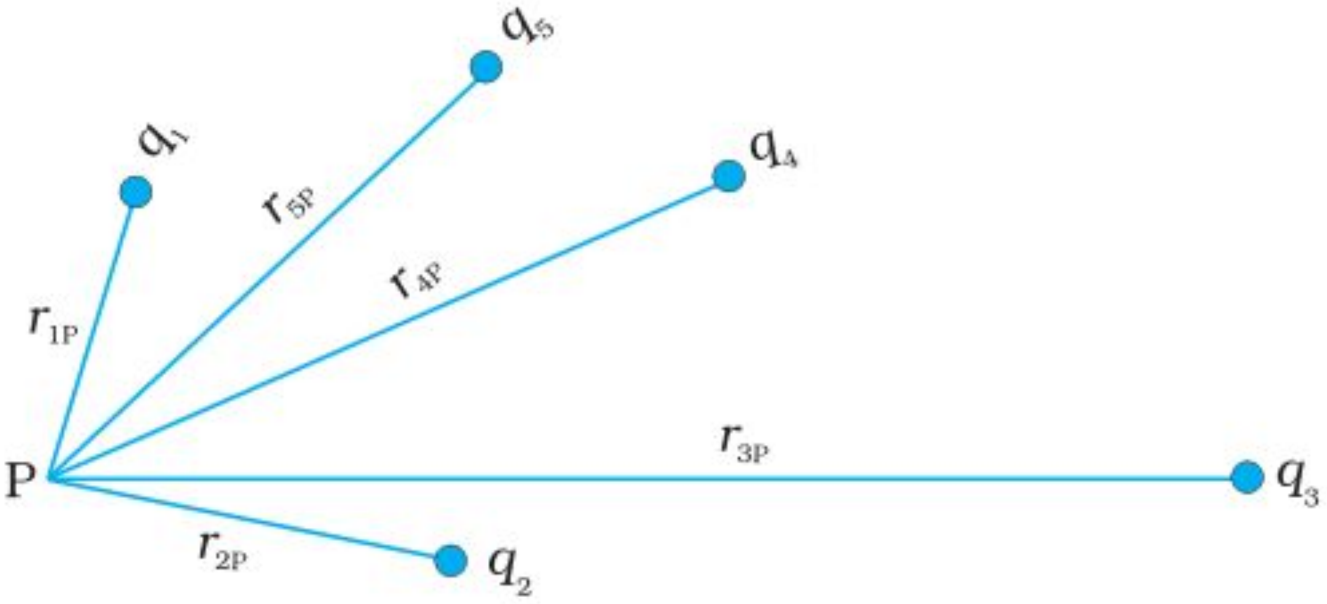
potential on the dipole a
by

$$V = \pm\frac{1}{4\pi\varepsilon_0}\frac{p}{r^2}$$

Positive sign for $\theta = 0$, negative sign for $\theta = \pi$.) The potential in the equatorial plane ($\theta=\pi/2$) is zero.

The potential due to a dipole depends not just on **r** but also on the **angle** between the position vector r and the dipole moment vector p

The electric dipole potential **falls off**, at large distance, as $\mathbf{1/r^2}$, not as $\mathbf{1/r}$, characteristic of the potential due to a single charge.



q_5
q_1
q_4
r_{5P}
r_{4P}
r_{1P}
r_{3P}
P
q_3
r_{2P}
q_2

POTENTIAL DUE TO A OF SYSTEM CHARGES

Consider a system of charges q_1, q_2,..., q_n with position vectors r_1, r_2,... relative to some origin . The potential V_1 at P due to the charge q_1 is

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{1P}}$$

where r_{1P} is the distance between q_1 and P.

Similarly, the potential V_2 at P due to q_2 and V_3 due to q_3 are given by

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_{2P}}, \quad V_3 = \frac{1}{4\pi\varepsilon_0}\frac{q_3}{r_{3P}}$$

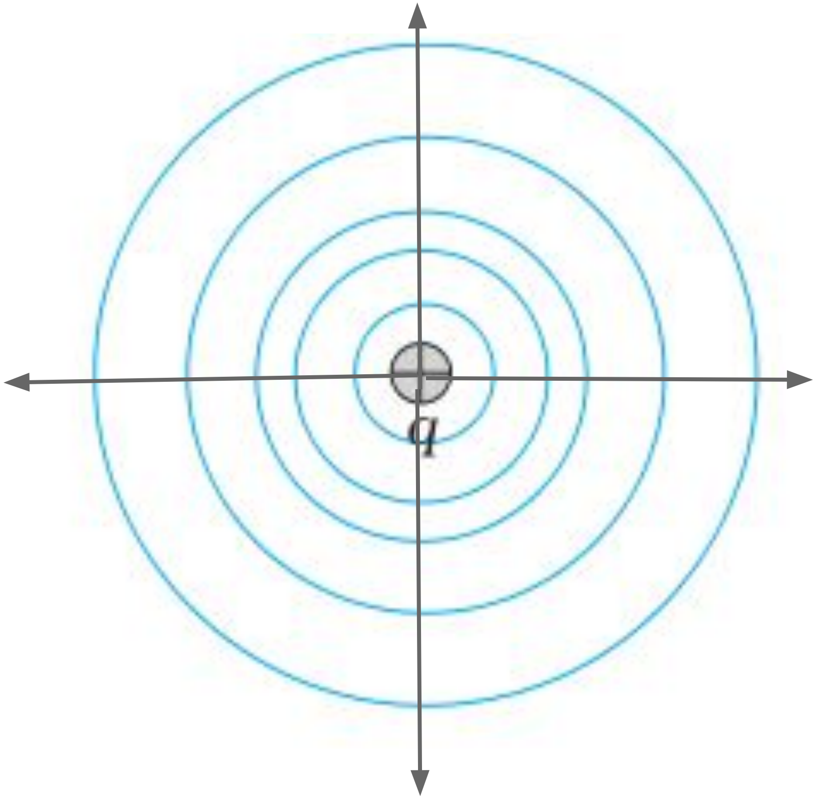
By the **superposition** principle, the potential V at P due to the total charge configuration is the **algebraic sum** of the potentials due to the **individual** charges $V = V_1 + V_2 + ... + V_n$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_{1P}} + \frac{q_2}{r_{2P}} + ... + \frac{q_n}{r_{nP}}\right)$$

EQUIPOTENTIAL SURFACES

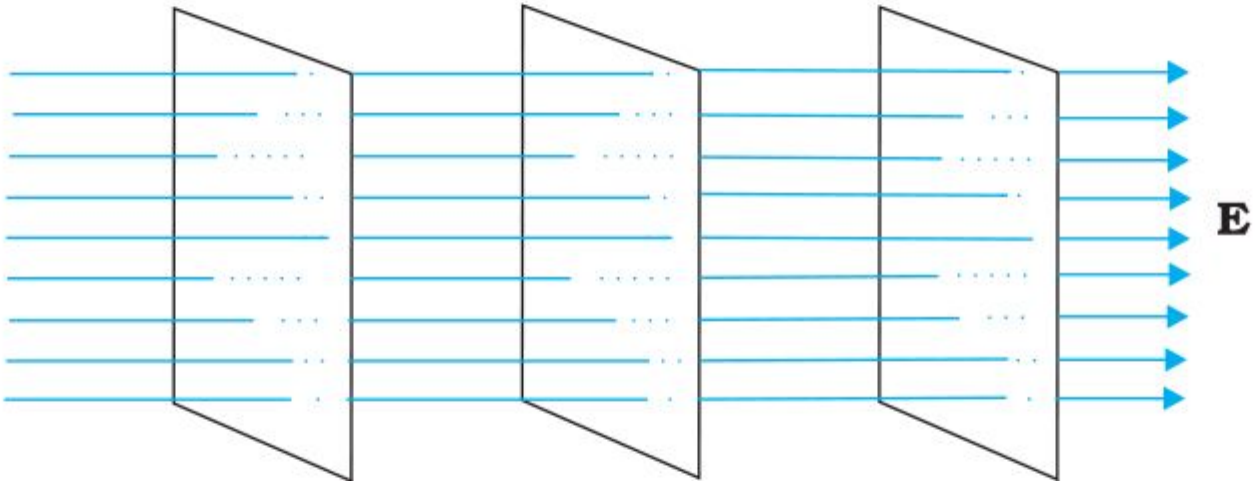
An equipotential surface is a surface with a constant value of potential at all points on the surface

For any charge configuration, equipotential surface through a **point** is **normal** to the **electric field** at that point



For a single charge the equipotential surfaces are spherical surfaces centered at the charges and the electric field lines are radial

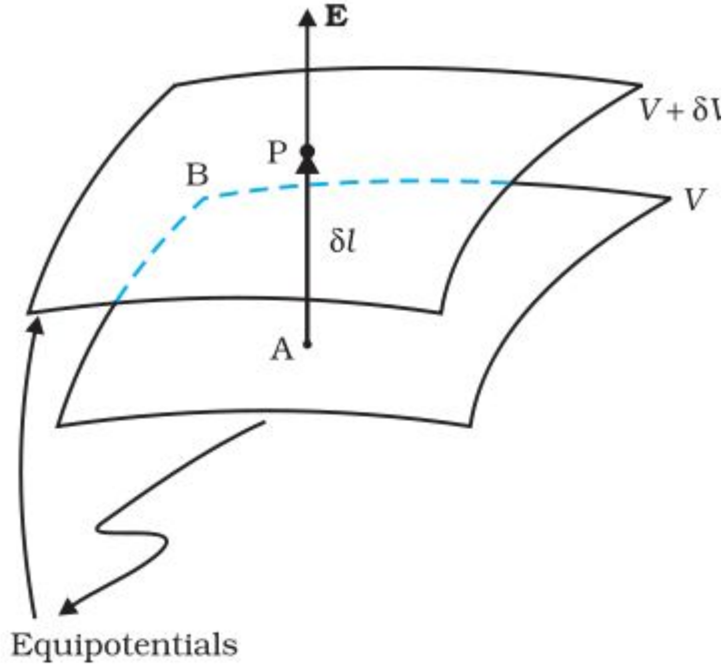
Equipotential surfaces for a uniform electric field.



Equipotential Surfaces of

a dipole and two identical positive charges.

Relation between field and potential



$$|\mathbf{E}| = -\frac{\delta V}{\delta l} = +\frac{|\delta V|}{\delta l}$$

relation between electric field and potential

(i) Electric field is in the **direction** in which the potential **decreases steepest**.

(ii)Electric field **magnitude** is given by the **change** in the magnitude of **potential** per **unit displacement** normal to the **equipotential** surface at the point.